**Math 250 Briggs 2.2 Definitions of Limits**

Objectives:

1) Use limit notation $f(x)$, $\lim_{x\to a} f(x) = L$
   a. Note that $\lim_{x\to a} f(x) = L$ is a limit of a single function, as opposed to the limit of an expression, like the slope of the secant line, $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$
   b. *L* is a finite, real number on the *y*-axis
   c. *a* is an *x*-coordinate
   d. As the *x*-coordinates get close to *a*, the *y*-coordinates get close to *L*.
2) Estimate limits using graphs.
3) Estimate limits using tables.
4) Use one-sided limit notation and concepts correctly
   a. Right-side limit: $\lim_{x\to a^+} f(x)$
   b. Left-side limit: $\lim_{x\to a^-} f(x)$
   c. Relationship of a limit $\lim_{x\to a} f(x)$ (two-sided) to the one-sided limits $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$.
      $\lim_{x\to a} f(x) = \lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$
   d. If the limit from the right does not equal the limit from the left, we say the limit (two-sided) does not exist.
   e. A limit with a + or – sign in the limit notation is always a one-sided limit.
   f. Limits without a + or – sign in the limit notation are two-sided, unless x approaches infinity. ("Limits at infinity", as x approaches infinity, are in a later section.)
5) Find limits of graphs with holes
   a. Recognize errors of graphs with holes when graphed on the graphing calculator
   b. Find limits when the function is not defined at a hole.
   c. Find limits when the function is defined piecewise at the hole.
   d. Notice that $\lim_{x\to a} f(x)$ is not necessarily equal to $f(a)$.
6) Three reasons that limits do not exist:
   a. $L \neq R$: The limit of the function from the left does not equal the limit of the function from the right: $\lim_{x\to a^+} f(x) \neq \lim_{x\to a^-} f(x)$
   b. Oscillation: A specific bizarre behavior near the point $x = a$ where the function has many different *y*-coordinates as the *x*-coordinates get close to *a*
   c. Unbounded: (*L* is not finite -- section 2.4)
7) Use the greatest integer function, also called the floor function, $\lfloor x \rfloor$.
   a. Evaluate
   b. Graph
   c. Find limits.

Examples

For each limit,

a) Graph the function and note whether the graph can be found accurately using a graphing calculator.

b) Estimate $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ using the graph and/or a table of values near $x = a$.

c) Use results to estimate $\lim_{x\to a} f(x)$.

d) Find $f(a)$.

e) Is $\lim_{x\to a} f(x) = f(a)$?

1) $\lim_{x\to 3}(x^2+2)$

2) $\lim_{x\to 3}\begin{cases}(x^2+2) & x\neq 3\\ \textit{undefined} & x=3\end{cases}$

3) $\lim_{x\to 3}\begin{cases}(x^2+2) & x\neq 3\\ 16 & x=3\end{cases}$

4) $\lim_{x\to 3}\begin{cases}(x^2+2) & x<3\\ x-4 & x>3\end{cases}$

5) $\lim_{x\to 0}\cos\left(\frac{1}{x}\right)$

6) $\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}$

7) $\lim_{x\to 2}\frac{x-2}{x^2-4}$

8) $\lim_{x\to 4}\left(\frac{\frac{x}{x+1}-\frac{4}{5}}{x-4}\right)$

9) $\lim_{x\to 1}\lfloor x\rfloor$

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Given a function $s(t)$, like a position function at time $t$, we could find

- limit of an expression: $\lim_{t_1 \to t_0} \frac{s(t_1) - s(t_0)}{t_1 - t_0}$

OR

- limit of just the function: $\lim_{t_1 \to t_0} s(t_1)$

If we change the name of the function to $f(x)$, we could find

- limit of an expression $\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

OR

- limit of just the function $\lim_{x \to a} f(x)$

GOAL: Use analytic methods to find limits of any function or expression involving a function.

EASIER GOAL: Find limit of a function only.

$$\lim_{x \to a} f(x)$$

FIRST STEP: Approximate the limit of a function only

$$\lim_{x \to a} f(x) \approx L$$

using either

a) graph $y = f(x)$

OR b) table

| $x$ | $f(x)$ |
|---|---|
| | |

or both.

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# Key facts about LIMITS

Notation is very specific and very important

$\lim_{x\to a} f(x)$ means "the limit of f as x appraches a".

⇒ Speak it correctly aloud and when you think it as you read.

$\lim_{x\to a} \boxed{\phantom{xx}}$ means "take the limit of expression in the box".

⇒ "$\lim_{x\to a}$" should never be written by itself. It's always "$\lim_{x\to a}$ (something)"

⇒ when writing the work, the limit notation should be written every step until you take the limit.

⇒ when writing the work, the limit notation should NOT be written after you take the limit.

⇒ If you want to refer to the end result, either write

limit is 3 ← Yes

OR $\lim_{x\to a} f(x) = 3$ ← Yes

⇒ Do NOT WRITE

$\lim_{x\to a} = 3$ ← NO.

you're missing what you took the limit of. ☹

The Answer to a limit is

- only one number
- a y-coordinate related to the graph.
- sometimes does not exist ⇒ write DNE and one of the three reasons a limit does not exist.

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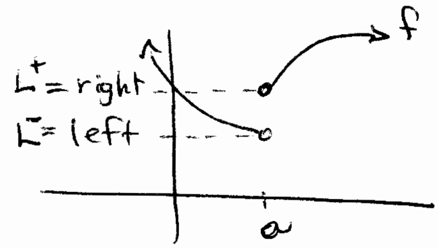
<u>One-sided limits</u> are a way to discuss behavior from left separately from behavior from right.

$$\lim_{x \to a} f(x) = \boxed{\begin{matrix} DNE \\ L^- \neq L^+ \end{matrix}}$$




For values of x which are smaller than a (left of a) the y-values are approaching L. We write this using superscript −.

$$\lim_{x \to a^-} f(x) = \boxed{L^-}$$

Similarly for values of x which are greater than c.

$$\lim_{x \to a^+} f(x) = \boxed{L^+}$$

$\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ are called <u>one-sided limits</u>.

The 2-sided limit $\lim_{x \to a} f(x)$ exists

if and only if both one-sided limits

$$\lim_{x \to a} f(x) = \lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$$

exist and are the same value.

$\lim_{x \to a^+} f(x)$ means one-sided limit from right
Consider only x-values greater than a

$\lim_{x \to a^-} f(x)$ means one-sided limit from left
Consider only x-values less than a

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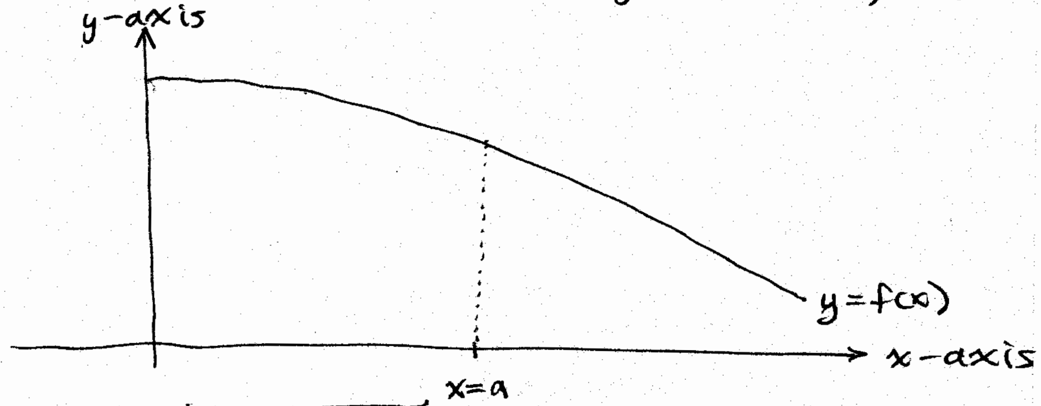
## $\boxed{\text{Approximating Limits Using a Graph}}$  $\lim_{x \to a} f(x)$

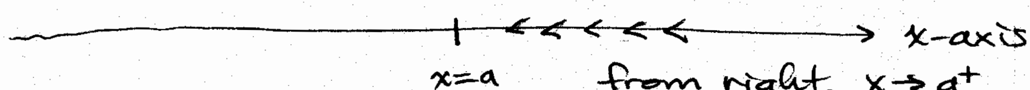
- Draw the graph $y=f(x)$ by hand or by GC
- Make sure graph is accurate near $x=a$.
- Consider both left of $x=a$ and right of $x=a$, separately first.



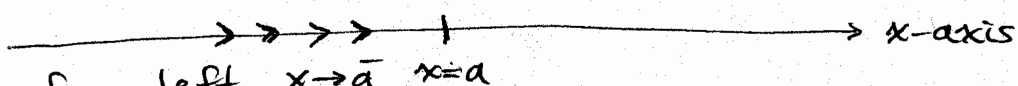

x-values to left of a are smaller than a. $x \to a^-$

x-values to right of a are larger than a $x \to a^+$

- Find the values of x and direction they move

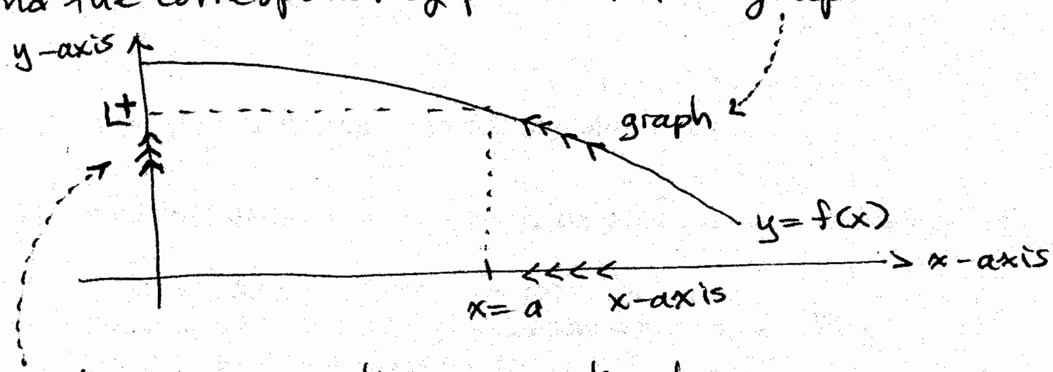



from right, $x \to a^+$
values of x get smaller
moving left toward $x=a$.




from left, $x \to a^-$
values of x get larger
moving right toward $x=a$.

- Find the corresponding part of the graph




- Find the corresponding y coordinates
- Identify the y-coordinate toward which the y-values trend.
- Repeat for left side, $L^-$
- Confirm the y-coordinates are equal: $L^+ = L^- = L$

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# $\boxed{\text{Approximating Limits Using Tables}}$ $\lim_{x\to a} f(x)$

↳ This is a numerical approach.

- Make a table of values of x that approach x=a from right

| $x$ | $f(x)$ |
|---|---|
| $a+0.1$ | |
| $a+0.01$ | |
| $a+0.001$ | |
| $a+0.0001$ | |
| $a+0.00001$ | |
| $a+0.000001$ | |
| | ↓ $L^+$ |

GC:
use $\boxed{\text{TBLSET}}$
Indpnt: $\boxed{\text{Ask}}$ ←-- keep Dep $\boxed{\text{Auto}}$
then $\boxed{\text{TABLE}}$
and type in the values of x
use ▶ and ▼/▲ to move down $Y_1$ column and see all available decimal places displayed at the bottom of GC screen

←-- If $\lim_{x\to a^+} f(x)$ exists, these y-coordinates will approach a value of y; we'll call it $L^+$.

- Repeat for values of x that approach x=a from left

| $x$ | $f(x)$ |
|---|---|
| $a-0.1$ | |
| $a-0.01$ | |
| $a-0.001$ | |
| $a-0.0001$ | |
| $a-0.00001$ | |
| $a-0.000001$ | |
| | ↓ $L^-$ |

←---- If $\lim_{x\to a^-} f(x)$ exists, these y-coordinates will approach a value of y; we'll call it $L^-$.

- Confirm that $L^+$ and $L^-$ are the same number, meaning

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$$

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$\boxed{\text{CAUTION: } f(a) \text{ versus } \lim_{x\to a} f(x)}$

The function value $f(a)$ is the y-coordinate when $x=a$.

- This is algebra. Subst $x=a$ into function and evaluate.
- Because $f(x)$ is a function, substituting $x=a$ results in at most one y-coordinate.
- Sometimes $f(a)$ is undefined; $x=a$ is not in domain of $f$.

The limit $\lim_{x\to a} f(x)$ is the value on the y axis that the y-coordinates approach as x approaches a.

- This is calculus. Estimate limit using graph or table; in later sections, we'll calculate limits exactly using analytic methods.
- Sometimes $\lim_{x\to a} f(x)$ does not exist.

The limit and function value are not equivalent concepts!

Here are five possible scenarios (but there are more...)

I. $f(a)=P$ and $\lim_{x\to a} f(x)=P$ $\longrightarrow$ $f(a)=\lim_{x\to a} f(x)$

II. $f(a)=P$ and $\lim_{x\to a} f(x)=Q\neq P$ $\longrightarrow$ $f(a)\neq \lim_{x\to a} f(x)$

III. $f(a)=P$ and $\lim_{x\to a} f(x)$ DNE* $\longrightarrow$ $f(a)\neq \lim_{x\to a} f(x)$

IV. $f(a)$ undefined and $\lim_{x\to a} f(x)=Q$ $\longrightarrow$ $f(a)\neq \lim_{x\to a} f(x)$

V. $f(a)$ undefined and $\lim_{x\to a} f(x)$ DNE* $\longrightarrow$ $f(a)\neq \lim_{x\to a} f(x)$

*DNE = Does Not Exist

**CAUTION:** $\lim_{x\to a} f(x)$ may not exist

- If $\lim_{x\to a} f(x)$ does not exist, we write "does not exist" OR "DNE".
- "Does not exist" is not the same as "undefined".
- $\lim_{x\to a} f(x)$ has a formal definition which we will study in a later chapter. The concept of a limit is always well-defined. It is never undefined. But.... sometimes there is no value which satisfies that definition, making the limit nonexistent.

## 3 REASONS A LIMIT COULD NOT EXIST

#1 The y-coordinate approaches a different value as $x \to a^-$ (from the left) from the y-coordinate it approaches as $x \to a^+$ (from the right).

$$\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x) \quad \text{or} \quad \text{"}L \neq R\text{"}$$

#2 The function values (y coordinates) as $x \to a$ oscillate among many y values; it does not settle on one y-value.

"OSCILLATION"

#3 The function values (y coordinates) as $x \to a$ get larger and larger ($y \to +\infty$) OR get smaller and smaller ($y \to -\infty$)
The limit is not finite; the limit is not bounded
(We study infinite limits more in a later section.)

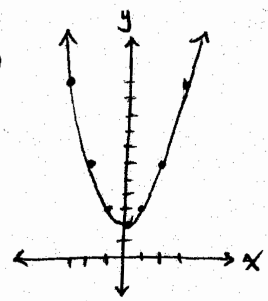
"UNBOUNDED"

**If a limit does not exist**

- ALWAYS write DNE or Does not exist
- ALWAYS write all reasons that apply.

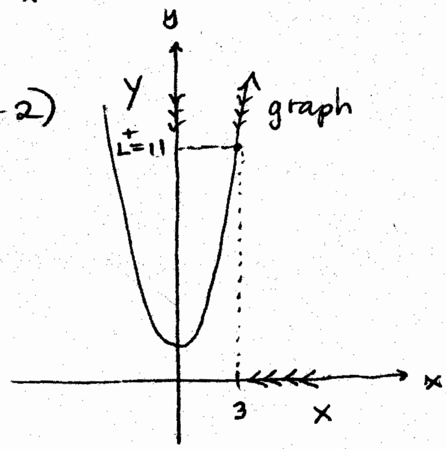
① $\lim_{x \to 3} (x^2+2)$

a)




Parabola has domain $(-\infty, \infty)$
GC graphs this accurately.

b) $\lim_{x \to 3^+} (x^2+2)$




$3^+$ means values of $x$ to the right of $x=3$, when a small positive amount is added to 3.

- find values of x and direction they move as limit is taken
- find corresponding portion of graph
- identify values of y and direction they move as limit is taken

Method 1

By graphical observation $L^+=11$

or $\lim_{x \to 3^+} (x^2+2) = \boxed{11}$

Method 2:

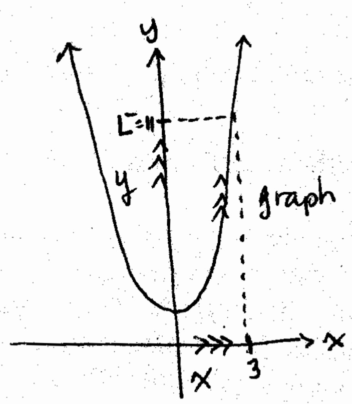
By numerical approximation, make a table as x approaches 3.

| x | 3.01 | 3.001 | 3.0001 | 3.00001 | 3.000001 |
|---|---|---|---|---|---|
| y | 11.0601 | 11.0060 | 11.0006 | 11.00006 | 11.000006 |

y values approach $\boxed{11}$.

$\lim_{x \to 3^-} (x^2+2)$

$= \boxed{11}$




Method 1

Method 2

| x | 2.9 | 2.99 | 2.999 | 2.9999 | 2.99999 |
|---|---|---|---|---|---|
| y | 10.41 | 10.9401 | 10.9940 | 10.99940 | 10.999940 |

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① c) From b) we saw that $\lim_{x\to3^+}(x^2+2) = \lim_{x\to3^-}(x^2+2) = 11$

Both the one-sided limits exist
Both the one-sided limits are the same.

Therefore $\lim_{x\to3}(x^2+2) = \boxed{11} = \lim_{x\to3^+}(x^2+2) = \lim_{x\to3^-}(x^2+2)$

d) f(a) means $f(3) \to 3^2+2 = \boxed{11}$.

e) Is $\lim_{x\to3} f(x) = f(3)$? $\boxed{\text{yes}}$.

$\lim_{x\to3}(x^2+2) = 11$

$f(3) = 11$

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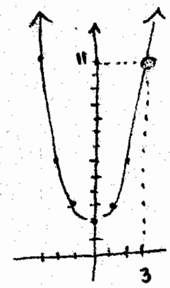
(2) $\lim\limits_{x\to 3} \begin{cases} x^2+2 & x \neq 3 \\ \text{undef} & x=3 \end{cases}$




a) GC can't graph this easily.

Let $f(x) = \begin{cases} x^2+2 & x \neq 3 \\ \text{undef} & x=3 \end{cases}$ to save writing!

b) $\lim\limits_{x\to 3^+} f(x) = \boxed{11}$ same as before

$\lim\limits_{x\to 3^-} f(x) = \boxed{11}$ same as before

c) $\lim\limits_{x\to 3} f(x) = \boxed{11}$ same as before!

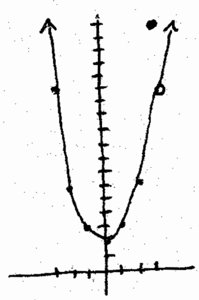
d) $f(3)$ is $\boxed{\text{undefined}}$

e) So $\lim\limits_{x\to 3} f(x) \neq f(3)$

$11 \neq$ undefined.

$\boxed{\text{NOT EQUAL}}$

(3) $\lim\limits_{x\to 3} \begin{cases} x^2+2 & x \neq 3 \\ 16 & x=3 \end{cases}$



a) GC can't graph this easily.

let's call this $g(x)$.

b) $\lim\limits_{x\to 3^+} g(x) = \boxed{11}$ same as before

$\lim\limits_{x\to 3^-} g(x) = \boxed{11}$ same as before

c) $\lim\limits_{x\to 3} g(x) = \boxed{11}$ same as before!

d) $g(3) = \boxed{16}$

e) So $\lim\limits_{x\to 3} g(x) \neq g(3)$

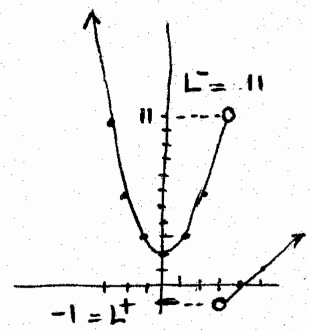
$11 \neq 16$

$\boxed{\text{NOT EQUAL}}$

Math 250

(4) $\lim_{x \to 3} \begin{cases} x^2+2 & x<3 \\ x-4 & x>3 \end{cases}$

a) GC can't graph this easily.




Let's call this one $h(x)$

b) $\lim_{x \to 3^+} h(x)$ is determined using <u>ONLY</u> the linear piece $x-4$

as $x \to 3^+$ (from right)

<u>Method 1</u> the linear graph has $y \to -1$

$$\lim_{x \to 3^+} h(x) = \lim_{x \to 3^+} (x-4) = \boxed{-1}$$

<u>Method 2:</u>

| x | 3.1 | 3.01 | 3.001 | 3.0001 | 3.00001 |
|---|---|---|---|---|---|
| y | −.9 | −.99 | −.999 | −.9999 | −.99999 |

$\lim_{x \to 3^-} h(x) = \boxed{11}$ same as before

c) $\lim_{x \to 3} h(x)$ $\boxed{\text{does not exist}}$ because $\lim_{x \to 3^+} h(x) \neq \lim_{x \to 3^-} h(x)$

If you like abbreviations: $\boxed{\begin{array}{l}\text{DNE} \\ L \neq R\end{array}}$

d) $h(3)$ is $\boxed{\text{undefined}}$ or $\boxed{\text{not defined}}$ because $x=3$ is not in either $x<3$ or $x>3$, and is not in the domain of $h(x)$.

e) $\lim_{x \to 3} h(x) \neq h(3)$

Does not exist $\neq$ undefined

$\boxed{\text{NOT EQUAL}}$

CAUTION: The words "undefined" and "does not exist" are NOT interchangeable.

The concept of a limit has a very specific definition, meaning that <u>limit</u> is always <u>defined</u>.

But the limit definition, when applied to a specific situation, may not give an answer — the result <u>does not exist</u>.

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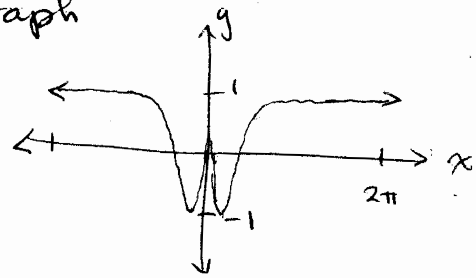
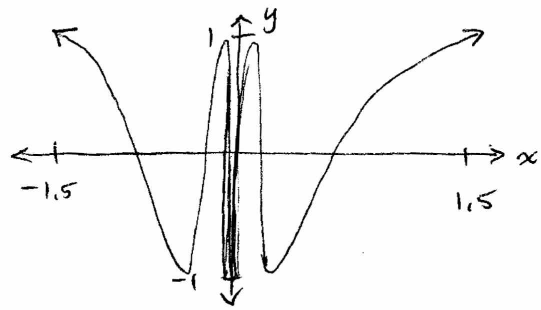
(5) $\lim_{x \to 0} \cos\left(\frac{1}{x}\right)$

a) Graph

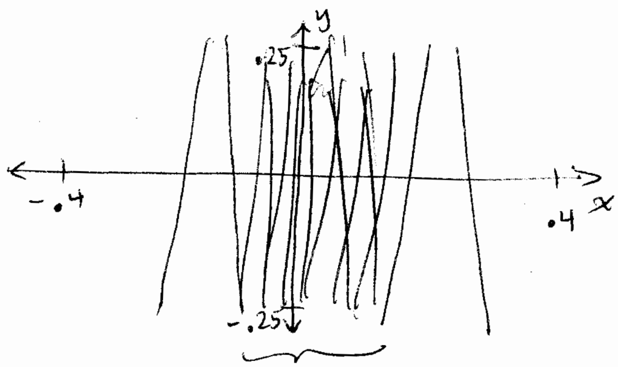



Use GC to look at graph.

ZOOM 7 = standard trig window




ZOOM IN 2 around (0,0), ENTER




ZOOM IN 2 around (0,0), ENTER

It gets worse and worse! Keep using ZOOM IN until you are convinced that this function is very, very naughty near $x = 0$.

b) $\lim_{x \to 0^+} f(x) = \boxed{\text{DNE, oscillation}}$

$\lim_{x \to 0^-} f(x) = \boxed{\text{DNE, oscillation}}$

c) $\lim_{x \to 0} f(x) = \boxed{\text{DNE, oscillation}}$

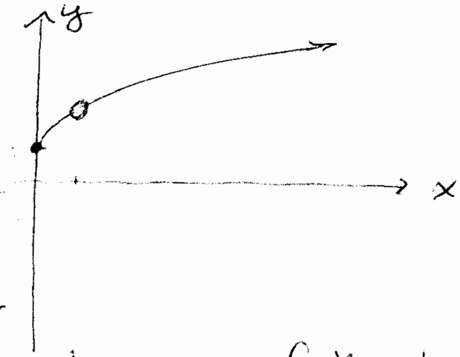
d) $f(0) = \cos\left(\frac{1}{0}\right) = \boxed{\text{undefined}}$

e) $\lim_{x \to 0} f(x) \neq f(0)$ $\boxed{\text{No}}$

divide by zero

M250 2.2

(6) $\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1}$




a) Graph $f(x) = \frac{x-1}{\sqrt{x}-1}$

No asymptote do before any w/ asymptote

GC does not gra y because of the hole

b) $\lim_{x \to 1^+} \frac{x-1}{\sqrt{x}-1}$ is not obvious from graph.

| 1 | 1.0001 | 1.001 | 1.01 | 1.1 |
|---|---|---|---|---|
| undefined | (2.000049999772) means 2.00000000 | 2.00049987 | 2.00498756 | 2.04880885 |

$\lim_{x \to 1^+} \frac{x-1}{\sqrt{x}-1} \approx \boxed{2}$ (8 decimal places)

$\lim_{x \to 1^-} \frac{x-1}{\sqrt{x}-1}$

| .9 | .99 | .999 | .9999 | 1 |
|---|---|---|---|---|
| 1.94868330 | 1.99498744 | 1.99949987 | 1.99995000 | undefined |

1.999949998

$\lim_{x \to 1^-} \frac{x-1}{\sqrt{x}-1} \approx \boxed{2}$

c) $\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1} = \boxed{2}$

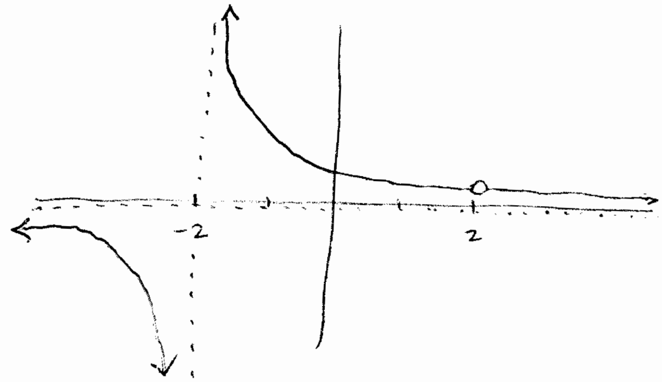
d) $f(1) = \frac{1-1}{\sqrt{1}-1} = \frac{0}{1}$ $\boxed{\text{undefined}}$

e) $\lim_{x \to 1} f(x) \neq f(1)$ $\boxed{\text{No}}$

M250 2.2

7. $\displaystyle\lim_{x\to 2} \frac{x-2}{x^2-4}$

a) Graph $f(x) = \dfrac{x-2}{x^2-4}$




GC does not graph this accurately because the hole at $x=2$ is not visible.
Some GCs also draw a vertical line at $x=-2$ where there is a vertical asymptote.
( $\boxed{\text{MODE}}$ $\boxed{\text{DOT}}$ sometimes fixes this )

b) $\displaystyle\lim_{x\to 2^+} \frac{x-2}{x^2-4}$ is not obvious from graph.

| | | | | |
|---|---|---|---|---|
| 2 | 2.0001 | 2.001 | 2.01 | 2.1 |
| undef | 0.2499937 | 0.2499375 | 0.2493766 | 0.2439024 |

GC TIP: Use $\boxed{\text{TBLSET}}$ independent = Ask. Then in $\boxed{\text{TABLE}}$ move cursor to y-coordinate and look at bottom of screen

$\displaystyle\lim_{x\to 2^+} \frac{x-2}{x^2-4} \approx \boxed{0.25}$

7 decimal places

$\displaystyle\lim_{x\to 2^-} \frac{x-2}{x^2-4}$

| | | | | |
|---|---|---|---|---|
| 1.9 | 1.99 | 1.999 | 1.9999 | 2 |
| 0.2564103 | 0.2506266 | 0.2500625 | 0.2500063 | undef |

$\displaystyle\lim_{x\to 2^-} \frac{x-2}{x^2-4} \approx \boxed{0.25}$

c) $\displaystyle\lim_{x\to 2} \frac{x-2}{x^2-4} = \lim_{x\to 2^+} \frac{x-2}{x^2-4} = \lim_{x\to 2^-} \frac{x-2}{x^2-4} = \boxed{0.25}$

d) $f(2) = \dfrac{2-2}{2^2-4} = \dfrac{0}{0}$ $\boxed{\text{undefined}}$

$\frac{0}{0}$ is called an indeterminate form, NEVER a final answer!

e) $\displaystyle\lim_{x\to 2} f(x) = .25$ but $f(2)$ undefined, so no, $f(2) \neq \displaystyle\lim_{x\to 2} f(x)$.

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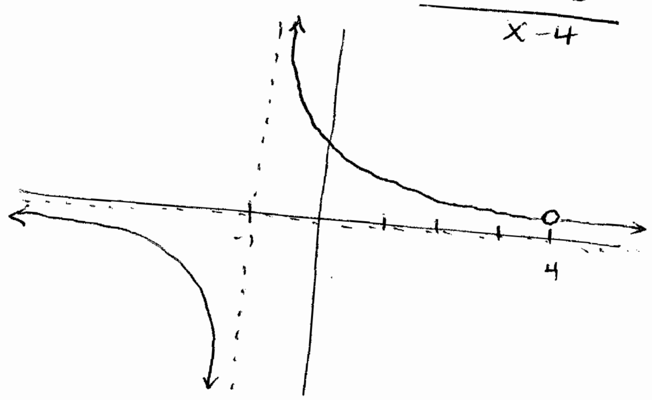
(8) $\lim_{x\to 4} \left( \dfrac{\dfrac{x}{x+1} - \dfrac{4}{5}}{x-4} \right)$

a) Graph $f(x) = \dfrac{\dfrac{x}{x+1} - \dfrac{4}{5}}{x-4}$

In GC, 3 sets of ( ) are required to communicate the intended order of operatio




$y_1 = (\ x/(x+1) - 4/5\ )/(x-4)$

↑ 1st numerator ↑ 2nd denom ↑ 3rd denom

GC does not graph this accurately because of the hole (and possibly the vertical asymptote)

b) $\lim_{x\to 4^+} f(x) \approx \boxed{0.}$

| 4 | 4.0001 | 4.001 | 4.01 | 4.1 |
|---|---|---|---|---|
| undefined | 0.04000 | 0.03999 | 0.03992 | 0.03922 |

$\lim_{x\to 4^-} f(x) \approx \boxed{0.04}$

| 3.9 | 3.99 | 3.999 | 3.9999 | 4 |
|---|---|---|---|---|
| 0.04082 | .04008 | .04001 | .04000 | undefined |

c) $\lim_{x\to 4} f(x) = \boxed{0.04}$

d) $f(4) = \boxed{\text{undefined}}$

e) $\lim_{x\to 4} f(x) \neq f(4)$ $\boxed{\text{NO}}$

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9. $\lim_{x\to 1} \lfloor x \rfloor$

a) $f(x) = \lfloor x \rfloor$ is the greatest integer function or floor function

For any real # $x$, $\lfloor x \rfloor$ is the greatest integer less than or equal to $x$.

ex: $f(0) = \lfloor 0 \rfloor = 0$ — 0 is an integer.

$f(1) = \lfloor 1 \rfloor = 1$

$f(-1) = \lfloor -1 \rfloor = -1$ — etc. for all integers

$f(\frac{1}{2}) = 0$ — $0 < \frac{1}{2}$ but $1 > \frac{1}{2}$.

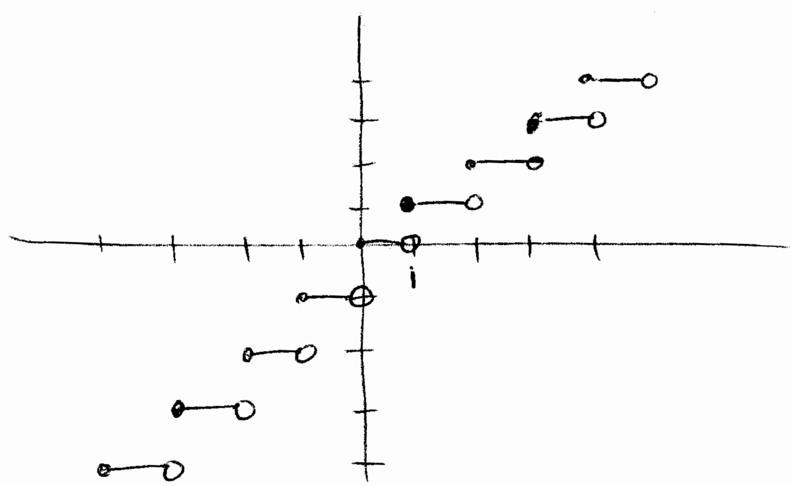
$f(\frac{3}{4}) = 0$

$f(\frac{7}{8}) = 0$

$f(1.1) = 1$

Graph in GC $y_1 = i\text{Part}(x)$

iPart is in MATH > NUM




This graph jumps at every integer

b) $\lim_{x\to 1^+} f(x) = \boxed{1}$

$\lim_{x\to 1^-} f(x) = \boxed{0}$

c) $\lim_{x\to 1} f(x) = \boxed{\text{DNE, } L \neq R}$

d) $f(1) = \boxed{1}$

e) $\lim_{x\to 1} f(x) \neq f(1)$

$\boxed{\text{No}}$

M250 2.2 Preview of 2.3

<u>Holes</u>: How do we know there is a hole, if the GC is giving an inaccurate graph?

Algebra!

If the expression can be simplified by
a) factor
b) cancel (divide out) common factor

then the root of the factor which cancels is the coordinate of the hole.

(7) $\dfrac{x-2}{x^2-4} = \dfrac{x-2}{(x-2)(x-4)} = \dfrac{1}{x-4}$ $x \neq 2$ hole.

$\dfrac{x-2}{x-2} = 1$

$x - 2 = 0$

$x = 2$

(6) $\dfrac{x-1}{\sqrt{x}-1} = \dfrac{(\sqrt{x})^2 - 1}{\sqrt{x} - 1} = \dfrac{(\sqrt{x}-1)(\sqrt{x}+1)}{(\sqrt{x}-1)}$

$= (\sqrt{x}+1)$ $x \neq 1$

$\sqrt{x} - 1 = 0$

$\sqrt{x} = 1$

$x = 1$

(8) $\dfrac{\dfrac{x}{x+1} - \dfrac{4}{5}}{\dfrac{x-4}{1}}$

GCF of all small denominators is $5(x+1)$.

M250 2.2

$$= \frac{\frac{x}{\cancel{(x+1)}} \cdot 5\cancel{(x+1)} - \frac{4}{\cancel{5}} \cdot \cancel{5}(x+1)}{\frac{x-4}{1} \cdot 5(x+1)}$$

mult each term top + bottom by GCF

$$\frac{5(x+1)}{5(x+1)} = 1$$

$$= \frac{5x - 4(x+1)}{5(x-4)(x+1)}$$

dist

$$= \frac{5x - 4x - 4}{5(x-4)(x+1)}$$

combine

$$= \frac{\cancel{x-4}}{5\cancel{(x-4)}(x+1)}$$

$x - 4 = 0$
$x = 4$ hole

$$= \frac{1}{5(x+1)} \quad x \neq 4$$

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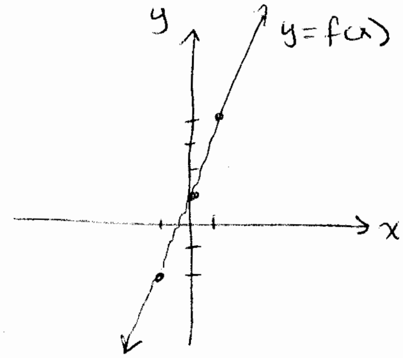
Extra Examples

Too simple

① $\lim_{x \to 1} (3x+1)$

a) Graph $f(x) = 3x+1$




GC does this correctly.
b/c f is defined $(-\infty, \infty)$

b) $\lim_{x \to 1^+} (3x+1)$ as $x \to 1$ from right, $y \to 4$

| | | | | |
|---|---|---|---|---|
| 1 | 1.0001 | 1.001 | 1.01 | 1.1 |
| | 4.0003 | 4.003 | 4.03 | 4.3 |

$\lim_{x \to 1^+} (3x+1) = \boxed{4}$

$\lim_{x \to 1^-} (3x+1)$ as $x \to 1$ from left, $y \to 4$

| | | | | |
|---|---|---|---|---|
| 0.9 | 0.99 | 0.999 | 0.9999 | 1 |
| 3.7 | 3.97 | 3.997 | 3.9997 | |

$\lim_{x \to 1^-} (3x+1) = \boxed{4}$

c) $\lim_{x \to 1} (3x+1) = \boxed{4} = \lim_{x \to 1^+} (3x+1) = \lim_{x \to 1^-} (3x+1)$

d) $f(1) = 3(1)+1 = \boxed{4}$

e) $\lim_{x \to 1} (3x+1) = f(1) = \boxed{4}$ yes.

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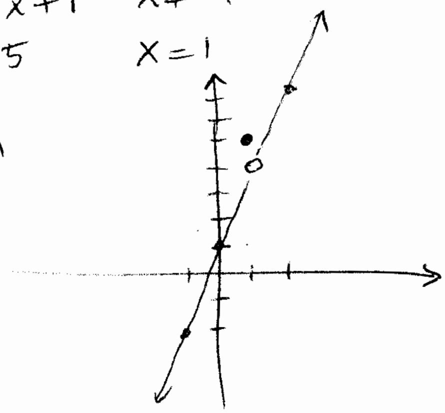
2 $\lim\limits_{x \to 1} \begin{cases} 3x+1 & x \neq 1 \\ 5 & x = 1 \end{cases}$

a) Graph



same tables as 1!

| | | | | |
|---|---|---|---|---|
| 1 | 1.0001 | 1.001 | 1.01 | 1.01 |
| irrelevant | 4.0003 | 4.003 | 4.03 | 4.3 |

| | | | | |
|---|---|---|---|---|
| .9 | .99 | .999 | .9999 | 1 |
| 3.7 | 3.97 | 3.997 | 3.9997 | irrelevant |

b) $\lim\limits_{x \to 1^+} f(x) = \boxed{4}$

$\lim\limits_{x \to 1^-} f(x) = \boxed{4}$

c) $\lim\limits_{x \to 1} f(x) = \boxed{4}$

d) $f(1) = \boxed{5}$

e) $\lim\limits_{x \to 1} f(x) \neq f(1)$ $\boxed{\text{NO}}$

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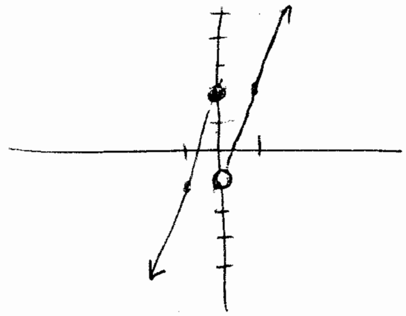
(3) $\lim\limits_{x\to 0} \begin{cases} 3x+2 & x \le 0 \\ 3x-1 & x > 0 \end{cases}$

a) Graph



This graph jumps at $x = 0$ from $(0,-1)$ to $(0,2)$

b) $\lim\limits_{x\to 0^+} f(x) = \boxed{-1}$

$\lim\limits_{x\to 0^-} f(x) = \boxed{2}$

c) $\lim\limits_{x\to 0} f(x)$ $\boxed{\text{does not exist}}$ because $\boxed{L \neq R}$

$\lim\limits_{x\to 0^-} f(x) \neq \lim\limits_{x\to 0^+} f(x)$.

d) $f(0) = 3(0)+2 = \boxed{2}$

e) $\lim\limits_{x\to 0} f(x) \neq f(0)$

DNE $\neq$ 2 $\boxed{\text{No}}$